Advanced Topics in Geometry E (MTH.B501)

Linear Ordinary Differential Equations

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Ordinary Differential Equations

$$\frac{d}{dt}\boldsymbol{x}(t) = f\big(t, \boldsymbol{x}(t)\big), \qquad \boldsymbol{x}(t_0) = \boldsymbol{x}_0 \qquad\qquad (*)$$

- Existence
- Uniqueness
- Regularity on initial conditions and parameters

Example

$$\frac{d}{dt}x(t) = f(t, x(t)) = \lambda x(t), \qquad x(0) = x_0.$$

Example

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} x_0 \cos \omega t + \frac{y_0}{\omega} \sin \omega t \\ -x_0 \omega \sin \omega t + y_0 \cos \omega t \end{pmatrix}$$

Example

$$\frac{dx}{dt} = 1 + x^2, \qquad x(0) = 0.$$