

Advanced Topics in Geometry E (MTH.B501)

Integrability Conditions

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Exercise 1-1

Problem (Ex. 1-1)

Find the maximal solution of the initial value problem

$$\frac{dx}{dt} = \lambda x(a - x), \quad x(0) = b,$$

where λ and a are positive constants, and b is a real number.

$$x(t) = \frac{ab}{(a - b)e^{-a\lambda t} + b}$$

Exercise 1-3

Problem (Ex. 1-3)

Find an explicit expression of a space curve $\gamma(s)$ parametrized by the arc-length s , whose curvature and torsion are $a/(1+s^2)$ and $b/(1+s^2)$, respectively, where a and b are constants.

$$\frac{d\mathcal{F}}{ds} = \frac{1}{1+s^2} \begin{pmatrix} 0 & -a & 0 \\ a & 0 & -b \\ 0 & b & 0 \end{pmatrix}$$