Advanced Topics in Geometry F (MTH.B502)

Local uniqueness of space forms

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Exercise 5-1

Problem (Ex. 5-1)

Prove that the sphere

$$S^n(c^2) = \left\{ \boldsymbol{x} \in \mathbb{R}^{n+1} \,;\, \langle \boldsymbol{x}, \boldsymbol{x} \rangle = \frac{1}{c^2} \right\}$$

of radius $1/c$ in the Eucidean $n+1$-space is of constant sectional curvature c^2.

Corrections on Proof of Theorem 5.5: $\frac{1}{c} \to c$

Exercise 5-2

Problem (Ex. 5-2)

Let $f\colon U \to \mathbb{R}^{n+1}$ *be an immersion defined on a domain* $U \subset \mathbb{R}^n$*, and* ν *a unit normal vector field. Take an orthornormal frame* $[\boldsymbol{e}_1, \dots, \boldsymbol{e}_n]$ *of the tangent bundle of* U*, and consider each* $\boldsymbol{e}_j$ *a map into* $\mathbb{R}^{n+1}$*. In addition, we consider* ν *an* $\mathbb{R}^{n+1}$*-valued function. Prove that*

$$d\nu = -\sum_j h^j \boldsymbol{e}_j, \qquad \textit{where} \qquad h^j := \langle d\boldsymbol{e}_j, \nu \rangle .$$