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Info. Sheet 3; Advanced Topics in Geometry E1 (MTH.B505)

Informations

- The classes on May 9th will be done in Friday's program. Next lecture is on May 16th.

- Due to the "golden week", the deadline of the hormework submission is changed to May 11 (Thu) 10:00 AM (JST).

Corrections

- Info. Sheet 2, line 2 of Q1: い幾何上 ⇒ 幾何上
- Lecture Note Page 5, line 6 (4th line of Definition 2.1): $U \cap V = \emptyset \Rightarrow U \cap V \neq \emptyset$
- Lecture Note Page 5, line 23: chart. . ⇒ chart. (Remove a period)
- Lecture Note Page 5, line 25 (2nd line of Definition 2.7): $(X_p)g \Rightarrow X_p(g)$
- Lecture Note Page 5, line -3 (1st line of Fact 2.9): $with \ \gamma(0) \Rightarrow with \ \gamma(0) = P$
- Lecture Note Page 6, line -5 (2nd line of Example 2.19): $\sum j = 0^n \Rightarrow \sum_{j=0}^{n}$
- Lecture Note Page 6, footnote 3: sabmanifolds ⇒ submanifolds
- Lecture Note Page 7, line 10: $(x^0, \ldots, {}^{n_1})^T \Rightarrow (x^0, \ldots, x^n)^T$
- Lecture Note Page 7, line 11: $M_q := S_1^{n-1}(k)) \Rightarrow S_1^{n-1}(k) := M_q$
- Lecture Note Page 7, line 16 (3rd line of Definition 2.21): $g_p(X_p, y_p) \Rightarrow g_p(X_p, Y_p)$
- Lecture Note Page 7, line 17 (4th line of Definition 2.21): sooth ⇒ smooth
- Lecture Note Page 8, line 3: $\varphi|_v \Rightarrow \varphi|_V$
- Lecture Note Page 8, line 9 (3rd line of Example 2.29): $\varphi := \psi \Rightarrow \varphi := \psi^{-1}$
- Lecture Note Page 8, line 14 (3rd line of Problem 2-1): Remove "●".

Students' comments

- 授業の資料，板書内容をすぐにアップロードいただき有難うございます．火曜日中に復習，課題提出ができるため効率よく学習できます． **Lecturer's comment** ご活用ください．
- I have some difficulty due to the English. It is the first time I am doing maths in English. **Lecturer's comment** Ok, as other students seems to feel so, I'll control the speed and expression more carefully.

Q and A

Q 1: Hyperbolic space $H^n(k)$ $(k < 0)$ や sphere $S^n(k)$ $(k > 0)$ の k の部分は変えても位相的な変化はないと思うのですが，微分幾何的には違いがあるのでしょうか．

A: 曲率の値が違います．

Q 2: What is the diffeomorphism? **A:** See Definition 2.25.

Q 3: What is the second axiom of countability?

A: It is the term in general topoly. To avoid "curious" examples, such a condition is included in the theory of manifold usually. However, as we will consider only submanifolds in $\mathbb{R}^n$ in this lecture, we do not need to mind this condition (which is satisfied automatically).

Q 4: Example 2.14: regarded as a smooth map $X: \mathbb{R}^n \to \mathbb{R}^n$ で $X_p: \mathbb{R}^n \to \mathbb{R}^n$ or $X: \mathbb{R}^n \to \mathbb{R}^n \times \mathbb{R}^n$ が正確だと思う．

A: ちょっと違います．$X_p \in T_p\mathbb{R}^n$ なので，接空間を $\mathbb{R}^n$ と同一視すれば $X_p \in \mathbb{R}^n$ であって $\mathbb{R}^n \to \mathbb{R}^n$ とはみなせません．Vector field は $p \in \mathbb{R}^n$ に対して $X_p \in T_p\mathbb{R}^n = \mathbb{R}^n$ の要素を対応させるので，$\mathbb{R}^n$ から $\mathbb{R}^n$ への写像とみなせます．力学や電磁気学で学んだベクトル場（具体的には重力の場や電場）は空間の点に対してベクトルを対応させていますね．