

# Advanced Topics in Geometry E1 (MTH.B505)

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2023/05/16

# Addendum

In Lemma 3.5, the assumption that  $\langle \cdot, \cdot \rangle$  to be non-degenerate is essential. Otherwise,  $m + r \leq n$  holds, in general.

Lemma

For an inner product  $\langle \cdot, \cdot \rangle$ , the subspace  $W_+$  (resp.  $W_-$ ) of  $V$  of dimension  $m$  (resp.  $r$ ) spans  $V$ , that is,  $V = W_+ \oplus W_-$ . In particular  $m + r = n$ .

$\langle x, y \rangle = 0 \quad \forall x, y$   
 $\Rightarrow r = 0$   
 $\langle x, y \rangle = x^T A y$   
( $\forall$  eigenvalues  $\neq 0$ )