Advanced Topics in Geometry F1 (MTH.B506)

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http://www.math.titech.ac.jp/~kotaro/class/2023/geom-f1/

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Important links:

- http://www.math.titech.ac.jp/~kotaro/class/2023/geom-f1
  (official web)
- http://www.official.kotaroy.com/class/2023/geom-f1   (a mirror)
- https://t2schola.titech.ac.jp/   (T2SCHOLA)

Lecture:

- ▶ Schedule: Tuesdays 10:45–12:25
- ▶ Venue: Main Building, M-143B
- ▶ Lecturer: Kotaro Yamada (Dept. Math.);
  kotaro@math.titech.ac.jp;
  Office: Room 231, the 2nd floor of the main building

Course Description

Definition and meanings of the "curvature" of Riemannian manifolds, especially those obtained as submanifolds of (pseudo) Euclidean space, are introduced.

Grading Policy:

- ► Graded by weekly homeworks.
- ► Each homework consists of (1) a problem on the topics in the lecture (up to 2 points), and (2) to present a question on the contents of the lecture, or to point out error(s) in the lecture note/the lecture (up to 3 points).
- ► Each homework should be submitted to T2SCHOLA by 10:00 on the following Thursday of the lecture, as an pdf file in the format of the homework sheet (which can be downloaded from the folder "Homework Sheet" on T2SCHOLA). Japanese is acceptable.
- ► Questions, requests and comments (and the answers, lecturer's comments) will be published on the following class.

Our Goal (of MTH.B505–506)

> **Theorem**
>
> *A __complete__ simply connected __Riemannian n-manifold__ of __constant sectional curvature k__ is isometric to*
>
> - *the Euclidean space $\mathbb{R}^n$ when $k = 0$,*
> - *the n-dimensional sphere $S^n(k) \subset \mathbb{R}^{n+1}$ if $k > 0$, and*
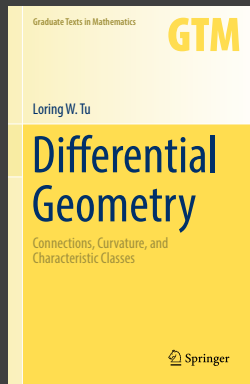> - *the n-dimensional hyperbolic space $H^n(k)$ if $k < 0$.*

cf. The fundamental theorem for surface theory

- sectional curvature ← Riemannian connection
- proof : solving a over determined system
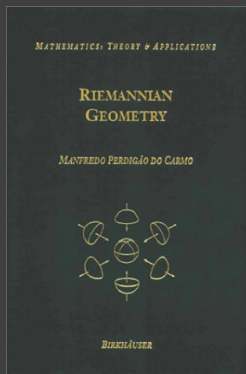 system of partial differential eq
  (curvature) → integrability condition

Our Goal (of MTH.B505–506)

- ✔ a simply-connected Riemannian n-manifold
- ✔ complete
- ▶ sectional curvature k
- ▶ isometric
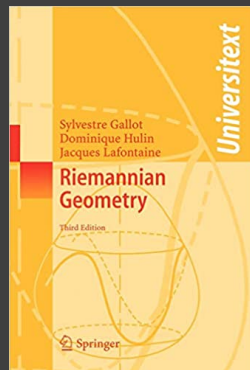- the Euclidean space $\mathbb{R}^n$
- the sphere S^n
- the hyperbolic space H^n

References

Tu

do Carmo

Gallot et. al.