Advanced Topics in Geometry F1 (MTH.B506)

Kotaro Yamada

kotaro@math.titech.ac.jp

http://www.math.titech.ac.jp/~kotaro/class/2023/geom-f1/

Tokyo Institute of Technology

2023/06/20

Q and A

Q: Are there an (not necessarily linear) ordinary differential equation such that all solution of it are defined on whole $\mathbb{R}$?

A: All solutions of the equation $\boxed{\ddot{x} = -\sin x}$ are defined on whole $\mathbb{R}$.

pendulum

$$\begin{cases} \dot{x} = y \\ \dot{y} = -\sin x \end{cases}$$

the conservation

law of mechanical energy.

x

$$\frac{1}{2}\dot{x}^2 - \cos x \ : \ \text{const.}$$

$$\ddot{x} + \sin x \cdot \dot{x} = 0$$

$$\left\{ \frac{1}{2}\left(\dot{x}^2 \right) - \cos x \right\}' = 0$$