

# Advanced Topics in Geometry F1 (MTH.B506)

Integrability Conditions

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## Problem 1-1

### Problem (Ex. 1-1)

*Find the maximal solution of the initial value problem*

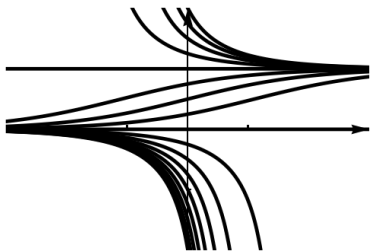
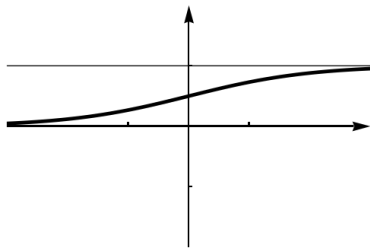
$$\frac{dx}{dt} = x(1 - x), \quad x(0) = a,$$

*where  $a$  is a real number.*

# The logistic equation

$$x' = x(1 - x), \quad x(0) = a$$

The Solution:  $x(t) = \frac{1}{\left(\frac{1}{a} - 1\right) e^{-t} + 1}$



## Problem 1-2

### Problem (Ex. 1-2)

*Find an explicit expression of a space curve  $\gamma(s)$  parametrized by the arc-length  $s$ , whose curvature  $\kappa$  and torsion  $\tau$  satisfy*

$$\kappa = \tau = \frac{1}{\sqrt{2}(1 + s^2)}.$$