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Info. Sheet 2; Advanced Topics in Geometry F1 (MTH.B506)

Informations

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Corrections

- Lecture Note, page 1, Fact 1.3: solutoins ⇒ solutions
- Lecture Note, page 6, the line next to the equation (1.17): $\widetilde{X}(t) = \widetilde{X}_{\mathrm{id},X_0,\tilde{\boldsymbol{\alpha}}}(t)$ ⇒ $\widetilde{X}(t) = \widetilde{X}_{0,X_0,\tilde{\boldsymbol{\alpha}}}(t)$
- Lecture Note, page 7, the first line of Theorem 1.17: *fnctions* ⇒ *functions*
- The blackboard `20230613-C-bb`, page 4: $\det X = \operatorname{tr} \widetilde{X} \frac{dX}{dt}$ ⇒ $\frac{d}{dt} \det X = \operatorname{tr} \widetilde{X} \frac{dX}{dt}$
- ODEs, PDEs のおそろいはうれしい. **Lecturer's comment** 特殊な状況ですが

Q 1: 線形でない常微分方程式で，非自明な解が $\mathbb{R}$（一般に $\mathbb{R}^n$）（山田注：解の定義域が $\mathbb{R}^n$ では常微分方程式にならないのでは？）全体で定義されるものはありますか？ Are there an (not necessarily linear) ordinary differential equation such that all solution of it are defined on whole $\mathbb{R}$?

A: All solution of the equation of pendulum $\ddot{x} = -\sin x$ are defined on whole $\mathbb{R}$.

Q 2: ODE (PDEs) の解の存在（一意性）でも使われる，積分曲線や平行移動，平坦性など，これら便利な定理の証明法（グロンウォールの不等式とか）が，キカ分野で活躍する場面はあるのでしょうか. Situations in geometry where "integral curves", "parallel translations", "flatness" and the method appeared in their proof, such as Gronwall's inequality play important roles.

A: The flatness is the key word in this lecture. In fact, the integrability condition is equivalent to flatness of a certain connection. An origin of integral curves and parallel translations are in differential geometry, and Gronwall's inequality is used implicitly in Section 1 in this lecture.