

# Advanced Topics in Geometry F1 (MTH.B506)

Curvature form

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## Problem 3-1

### Problem

Let  $\{e_j\}$  and  $\{v_j\}$  be two orthonormal frames on a domain  $U$  of a Riemannian  $n$ -manifold  $M$ , which are related as

$$[e_1, \dots, e_n] = [v_1, \dots, v_n]\Theta.$$

Show that the connection forms  $\Omega$  of  $\{e_j\}$  and  $\Lambda$  of  $\{v_j\}$  satisfy  $\Omega = \Theta^{-1}\Lambda\Theta + \Theta^{-1}d\Theta$ .

## Problem 3-2

### Problem

Let  $\mathbb{R}_1^3$  be the 3-dimensional Lorentz-Minkowski space and let  $H^2(-1)$  the hyperbolic 2-space of constant curvature  $-1$

- 1 Verify that

$$f(u, v) = (\cosh u, \cos v \sinh u, \sin v \sinh u)$$

gives a local coordinates on  $U := H^2(-1) \setminus \{(1, 0, 0)\}$ , and

$$e_1 := (\sinh u, \cos v \cosh u, \sin v \cosh u),$$

$$e_2 := (0, -\sin v, \cos v)$$

forms an orthonormal frame on  $U$ .

- 2 Compute the connection form(s) with respect to the orthonormal frame  $\{e_1, e_2\}$ .