July 04, 2023
Kotaro Yamada
kotaro@math.titech.ac.jp

# Info. Sheet 4; Advanced Topics in Geometry F1 (MTH.B506)

## Corrections

- Lecture note, page 14, line 5: $\mathcal{F}(M)$ as $(Xf)(\mathrm{P}) = X_{\mathrm{P}}f \Rightarrow \mathcal{F}(M)$ as $(Xf)(p) = X_p f$
- Lecture note, page 14, line 16: For each p $\in M \Rightarrow$ For each $p \in M$
- Lecture note, page 14, line 2 of Lemma 3.1:
$$\left(\frac{\partial}{\partial}x^j\right)_p \qquad \Rightarrow \qquad \left(\frac{\partial}{\partial x^j}\right)_p$$
- Lecture note, page 14, equation (3.7): $\mathcal{F}(M \Rightarrow \mathcal{F}(M)$
- Lecture note, page 14, line $-10$:
$$T_p^*M \otimes T_p^*M \otimes T_p^*M \colon T_pM \qquad \Rightarrow \qquad T_p^*M \otimes T_p^*M \otimes T_p^*M$$
- Lecture Note, page 16, equation (3.12):
$$X\langle Y, Z\rangle = \langle \nabla_X Y, Z\rangle + \langle X, \nabla_X Z\rangle \qquad \Rightarrow \qquad X\langle Y, Z\rangle = \langle \nabla_X Y, Z\rangle + \langle Y, \nabla_X Z\rangle$$
- Lecture Note, page 16, Definition 3.7: *Levi-Covet* $\Rightarrow$ *Levi-Civita*
- Lecture Note, page 16, Remark 3.9: *amine connection* $\Rightarrow$ *affine connection*
- Lecture note, page 18, Exercise 3-2 (1): Verify that gives $\Rightarrow$
Verify that
$$\boldsymbol{f}(u, v) := (\cosh u, \cos v \sinh u, \sin v \sinh u)$$
gives

## Q and A

**Q 1:** 接続形式はフレームに大きく依存するため，少々扱いにくい（扱う際に注意を払う）ように感じますが，先生はどのように感じますか？ Dependence of connection forms on choice of frames.

**A:** Frame dependence of the connection forms is essential property of connections (as kown as “Gauge transformations”), which is expressed as in Exercise 3-1 for connection forms, as well as the coordinate change of the Christoffel symbols. See also Theorem 4.11 and Exersise 4-1.