Advanced Topics in Geometry F1 (MTH.B506)

Sectional Curvature

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Problem 4-1

Problem

Consider a Riemannian metric $g = dr^2 + \{\varphi(r)\}^2 \, d\theta^2$ on $U := \{(r, \theta) \, ; \, 0 < r < r_0, -\pi < \theta < \pi\}$, where $r_0 \in (0, +\infty]$ and φ is a positive smooth function defined on $(0, r_0)$ with

$$\lim_{r \to +0} \varphi(r) = 0, \qquad \lim_{r \to +0} \varphi'(r) = 1.$$

Find a function φ such that (U, g) is flat.
Hint: $[\partial/\partial r, (1/\varphi)\partial/\partial\theta]$ is an orthonormal frame.

Problem 4-2

Problem

Compute the curvature form of $H^2(-1)$ with respect to an orthonormal frame $[\boldsymbol{e}_1, \boldsymbol{e}_2]$ as in Exercise 3-2.

$$\boldsymbol{f} = (\cosh u, \cos v \sinh u, \sin v \sinh u) \colon (0, \infty) \times (-\pi, \pi) \to \mathbb{R}^3_1$$
$$\boldsymbol{e}_1 = (\sinh u, \cos v \cosh u, \sin v \cosh u),$$
$$\boldsymbol{e}_2 = (0, -\sin v, \cos v)$$