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Info. Sheet 3; Advanced Topics in Geometry A1 (MTH.B405)

Informations

- Sorry that the uploaded files of the blackboards on April 25 was incorrect. I've uploaded the correct versions.

Corrections

- Lecture note, page 6, line 1: $\tan\frac{t^2}{1} \Rightarrow \tan\frac{t^2}{2}$
- Lecture note, page 7, line 13: $\sqrt{\lambda_1^2+\dots+\lambda_n^2} \Rightarrow \sqrt{\lambda_1+\dots+\lambda_n}$
- Lecture note, page 7, line 22: Leibnitz ⇒ Leibniz
- Lecture note, page 8, footnote 6: lienar ⇒ linear
- Lecture note, page 9, line 12: $a > 0$such that ⇒ $a > t_0$ such that
- Lecture note, page 9, line 17: $\leqq \dfrac{e^{k(t-t_0)}}{|k|} \Rightarrow \leqq \dfrac{e^{kt}}{|k|}$
- Lecture note, page 10, eq. (2.15): $X(t) = \Rightarrow X(t) :=$
- Blackboard B, page 13, the first line of Theorem 2.17: fnctions ⇒ functions

Students' comments

- 行列表示された線型常備分方程式の解に関する議論から空間曲線の基本定理までの証明を久しぶりに精読すると，よい復習になりました．
  It was a good review to read the proofs precisely, from the discussion on the solution of matrix-valued linear ordinary equations to the fundamental theorem of space curves.

  Lecturer's comment Really?

- This is and interesting and usual course. There is no other request.

  Lecturer's comment Sure.

- 録画を公開してくれると助かります．英語を聞いてもわからないので翻訳ツールが使いやすくて楽です．ありがとうございます．
  The recordings are helpful, because the translation tool help me to understand English. Thanks.

  Lecturer's comment You're welcome.

- Exercise 2-3 の計算は大変でした．なにかよりよい方法があるかは気になります．
  The calculations in Exercise 2-3 were very hard. I wonder if there is a better way.

  Lecturer's comment I'll explain it in today's lecture.

Q and A

Q 1: $\frac{dx}{dt} = 1 + x^2$ を初期条件 $x\left(\frac{\pi}{2}\right) = 0$ で解いてみたら，多分 $x(t) = -\frac{\cos t}{\sin t}$ $(t \in (0, \pi))$ になりました．この定義域の幅 π は初期値 $x(0) = 0$ のときと同じ幅です．同じ微分方程式に対する極大解の幅は初期値を変えても同じだったりしませんか？（予想は特別な値のときだけ変化することもありそう．）
Solving $\frac{dx}{dt} = 1 + x^2$ with initial condition $x\left(\frac{\pi}{2}\right) = 0$, I got the solution $x(t) = -\frac{\cos t}{\sin t}$ $(t \in (0, \pi))$. The width of this domain π is the same as the initial value $x(0) = 0$. Isn't the width of the maximal solution for the same differential equation the same for different initial values? (The expectation may change only for special values.)

A: No. The equation $\frac{dx}{dt} = t(1 + x^2)$ is a counterexample. In fact, the solution $x(t) = \tan \frac{t^2}{2}$ for the initial condition $x(0) = 0$ is defined on $(-\sqrt{\pi}, \sqrt{\pi})$. On the other hand, the solution $x(t) = \tan\left(\frac{t^2}{2} + \frac{\pi}{4}\right)$ for $x(0) = 1$ is defined on $(-\sqrt{\pi/2}, \sqrt{\pi}2)$.

Q 2: たとえば 2-1 では x の定義域を与えずに解いているが,「定義域がわからない（定まっていない）が，微分方程式がわかる関数」を知りたくなることがあるだろうか．それはどんなシチュエーションなのか（物理学などの例だと実感しやすい)．
In Exercise 2-1, for example, the equation is solved without giving the domain of x. Is there a situation that one wants to know "a function whose domain is not known (not determined) but whose differential equation is known"? Examples in physics are easy to understand.

A: Usually, the domain of definition of a solution of a (non-linear) differential equation is determined *a posteriori*. The equation of geodesic on surfaces, or Riemannian manifolds, are such examples. A solution of Ricci flow equation (although it is not an ordinary differential equation) is also usually diverges.