Advanced Topics in Geometry B1 (MTH.B406)

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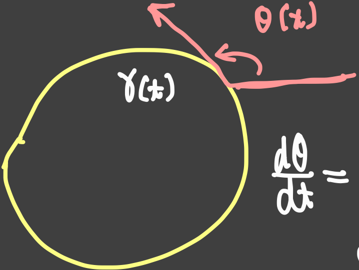
Q and A

立体角

Q: The integral of the curvature of a curve is an angle (exterior angle), but is there any name for the integral of the Gaussian curvature of a surface? It is different from solid angle, isn't it? (For example, how can I say that each vertex of a regular polyhedron with n vertices has an "exterior angle" of $4\pi/n$?)

$\mathbb{R}^2$

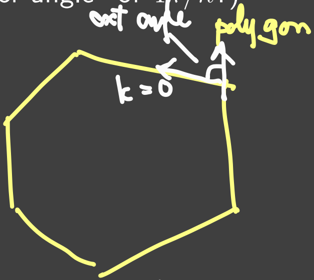


$$\frac{d\theta}{dt} = \kappa$$

curvature

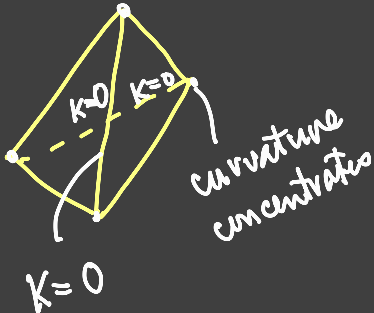
$$\underline{\int \kappa \, dt = 2\pi}$$

$(\text{if } |\dot{\gamma}(t)| = 1)$



$$\underline{\sum \text{exterior angles} = 2\pi}$$

convex polyhedron



sphere

Gauss Bonnet

$$* \int K \, dA = 4\pi$$
$$= 2 \text{ (Euler number)}$$

How can one define curvature (measure) at vertices?

To keep Gauss-Bonnet formula.

Q and A

Gauss. curv. K -1 : a pseudospherical surf

Q: Isn't a surface with constant Gaussian curvature of 1 called a spherical surface?

the sphere is a special surface in spherical surface.

A: Yes. Sphere : $K = 1$

special pseudospherical surface

Q: If Example 2.7 is "Beltarmi's" pseudosphere, does that mean there are other pseudospheres? What characterizes a pseudosphere from other pseudosurfaces?

- We did not define "pseudospheres".

Q and A

Q: As the hyperbolic plane is "hyperbolic", hyperbolic functions appear in the Pythagorean formula and Beltrami's pseudosphere. Is this because hyperbolic functions appear in the solution of differential equations, as in problem 2-1? (I thought this was in contrast to the fact that trigonometric functions appear when K is positive.)

$\exists$ many cases of correspondence

" $\cos \longrightarrow \cosh$ "

$\sin \longrightarrow \sinh$

$K>0$ $\quad K<0$

(e.g.) Pythagorean formulas

: Lecture 7,

another example.