Advanced Topics in Geometry B1 (MTH.B406)

Asymptotic Chebyshev nets

Kotaro Yamada
kotaro@math.sci.isct.ac.jp
http://www.official.kotaroy.com/class/2025/geom-b1

Institute of Science Tokyo

2025/06/27

Exercise 2-1

Problem

Let $\gamma(t) = \big(x(t), z(t)\big)$ $(\gamma \in I)$ *be a parametrized curve on the* xz*-plane satisfying*

$$\big(x'(t)\big)^2 + \big(z'(t)\big)^2 = 1 \qquad (t \in I), \tag{$*$}$$

where $I \subset \mathbb{R}$ *is an interval. Consider a surface*

$$p_\gamma(s,t) := \big(x(t)\cos s, x(t)\sin s, z(t)\big),$$

which is a surface of revolution of profile curve γ*.*

1. *Show that* p_γ *is pseudospherical if and only if* $x'' = x$ *holds.*
2. *Can one choose* $I = \mathbb{R}$*?*

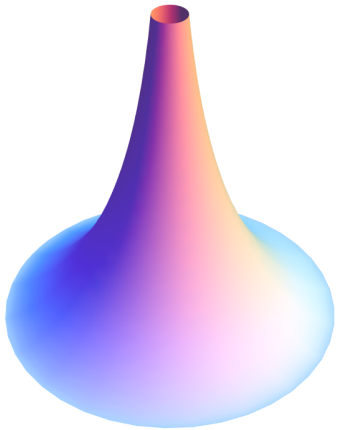
Exercise 2-1

$x'' = x \Rightarrow x = A\cosh s + B\sinh s$:

- $A^2 - B^2 > 0$, $x = \pm\sqrt{A^2 - B^2}\cosh(s + \alpha)$,
- $A^2 - B^2 < 0$, $x = \pm\sqrt{B^2 - A^2}\sinh(s + \alpha)$,
- $A^2 = B^2$, $x = \pm Ae^{\pm s}$.

Exercise 2-1

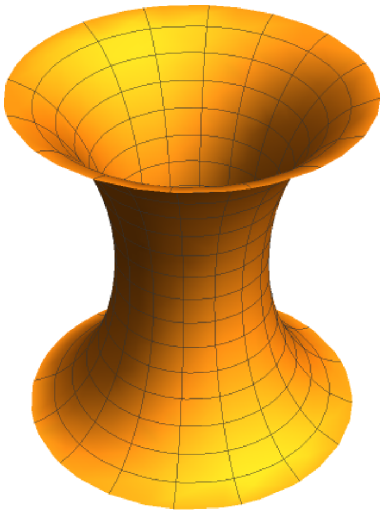
$x = e^{-s}$



$z = -\sqrt{1 - e^{-2s}} + \cosh^{-1} e^{s}$

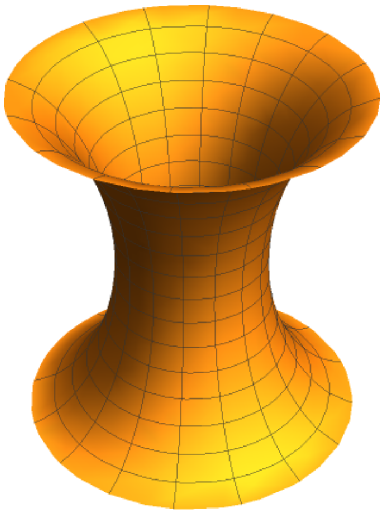
Exercise 2-1

$x = A\cosh s$



Exercise 2-1

$x = A \sinh s$



Exercise 2-1

2. Can one choose $I = \mathbb{R}$?

Exercise 2-1

Q: Does the fact that I can only be defined on a finite interval mean that the looped $\gamma(t)$ cannot be? I might be wrong.

Exercise 2-2

Problem

Let a and b be real numbers with $a \neq 0$. Compute the Gaussian curvature of the surface

$$p(u, v) = a(\operatorname{sech} v \cos u, \operatorname{sech} v \sin u, v - \tanh v) + b(0, 0, u).$$

Exercise 2-2