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Info. Sheet 3; Advanced Topics in Geometry B1 (MTH.B406)

Corrections

- Blackboard/Handout C, Exersise 2-1: prfile ⇒ profile
- Lecture note, page 5, bottom: independent on choice of ⇒ independent on the choice
- Lecture note, page 6, Eq. (2.8): $p \circ \nu(t) \Rightarrow \nu \circ \gamma(t)$
- Lecture note, page 6, line 12: which is called the **edge** ⇒ which are called **edges**
- Lecture note, page 7, line 10: In this seance ⇒ In this sence
- Lecture note, page 7, line 11: latrer ⇒ later
- Lecture note, page 7, line −2: $\bigl((u,v) \in (0,+\infty)\times(-\pi,\pi)\bigr) \Rightarrow \bigl((u,v) \in (-\pi,\pi)\times(0,+\infty)\bigr)$
- Lecture note, page 8, Exercise 2-1-(*):

$$\bigl(x'(t)\bigr)^2 + (z'(t))^2 = 1 \qquad (t \in I)$$
$$\Rightarrow \quad \bigl(x'(t)\bigr)^2 + (z'(t))^2 = 1, \qquad x(t) > 0, \qquad (t \in I),$$

- Lecture note, page 8, Exercise 2-1-(1): $z'' = z \Rightarrow x'' = x$
- Lecture note, page 8, Exercise 2-1-(1), Hint: $x'x'' + y'y'' = 0 \Rightarrow x'x'' + z'z'' = 0$

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Q 1: 曲線の曲率の積分は角度（外角）ですが，曲面のガウス曲率の積分になにか名前はありますか？立体角とは違いますよね．（たとえば n 個の頂点を持つ正多面体の各頂点は $4\pi/n$ の“外角”を持っているということはどう言えばよいのでしょうか？）
The integral of the curvature of a curve is an angle (exterior angle), but is there any name for the integral of the Gaussian curvature of a surface? It is different from solid angle, isn't it? (For example, how can I say that each vertex of a regular polyhedron with n vertices has an "exterior angle" of $4\pi/n$?)

A: For example, what curvature must be concentrated at the vertices of a convex polyhedron for Gauss-Bonnet's theorem to hold?

Q 2: ガウス曲率が恒等的に 1 の曲面を spherical surface と呼ぶことはないんですか??
Isn't a surface with constant Gaussian curvature of 1 called a spherical surface?

A: Yes, such surfaces are called "spherical surfaces".

Q 3: I が有限区間上でしか定義できないということはループした $\gamma(t)$ をとれないということでしょうか？ 見当違いかもしれません．
Does the fact that I can only be defined on a finite interval mean that the looped $\gamma(t)$ cannot be? I might be wrong.

A: No, it cannot be. If one can take a closed curve γ, then it is considered as a curve defined on $\mathbb{R}$, which is the universal cover of S^1.

Q 4: 双曲平面が「双曲」という通り，双曲線関数がピタゴラスの公式や Beltrami の pseudosphere に現れますが，これは問題 2-1 のように微分方程式の解に双曲線関数が現れるからでしょうか？（K が正のとき三角関数が現れることと対比していると思いました）
As the hyperbolic plane is "hyperbolic", hyperbolic functions appear in the Pythagorean formula and Beltrami's pseudosphere. Is this because hyperbolic functions appear in the solution of differential equations, as in problem 2-1? (I thought this was in contrast to the fact that trigonometric functions appear when K is positive.)

A: It's hard to say, but often when you change trigonometric functions to hyperbolic functions, you get "hyperbolic" figures.

Q 5: Regarding the exercise 2.2, I am curious what the surface looks like, as it be a slight deformation of pseudosphere as shown in example 2.7. When $b = 0$, I guess it is only a kind of dilatation? What happens when $b \neq 0$?

A: I'll explain it on the lecture.

Q 6: If Example 2.7 is “Beltarmi’s” pseudosphere, does that mean there are other pseudospheres? What characterizes a pseudosphere from other pseudosurfaces?

A: The word “pseudosurfaces” is incorrect. “Pseudospherical surfaces” are defined, but pseudosphere is still an undefined word. Then “Beltrami’s pseudosphere” itself is a special example of pseudospherical surfaces. In addition, surfaces as Exercise 2.7 is called “Dini’s pseudosphere”.