Advanced Topics in Geometry B1 (MTH.B406)

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Student's comments

- The calculations became too complicated and I could not solve all the problems.
  Comment: Sorry.

Q and A

$\theta \in (-\pi, 0)$

The asymptotic Chebyshev net:

$\to (\xi, \eta) \mapsto (-\eta, \xi)$

$$ds^2 = d\xi^2 + 2\cos\theta\, d\xi\, d\eta + d\eta^2, \qquad II = k \sin\theta\, d\xi\, d\eta.$$

$\theta \to \pi + \theta$.

Q: Why do we assume $0 < \theta(\xi, \eta) < \pi$?

To guarantee the 1st fundamental matrix

$$I = \begin{pmatrix} 1 & \cos\theta \\ \cos\theta & 1 \end{pmatrix}$$

is non-singular.

$\Leftrightarrow$ regular surface.

$\det I = 1 - \cos^2\theta = \sin^2\theta$

$\theta \not\equiv 0 \mod \pi$

$\theta \in (n\pi, (n+1)\pi)$

by a suitable change of coord

Q and A

The asymptotic Chebyshev net:

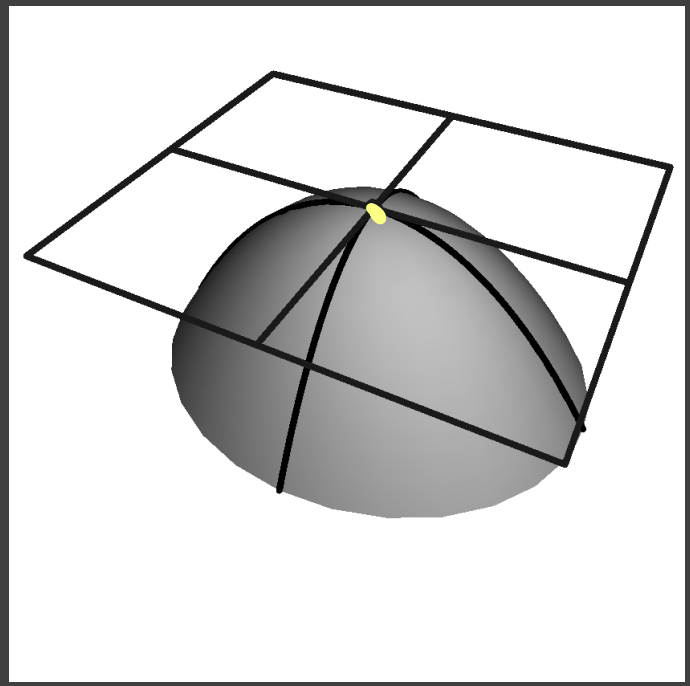
$$ds^2 = d\xi^2 + 2\cos\theta\, d\xi\, d\eta + d\eta^2, \qquad II = k\sin\theta\, d\xi\, d\eta.$$

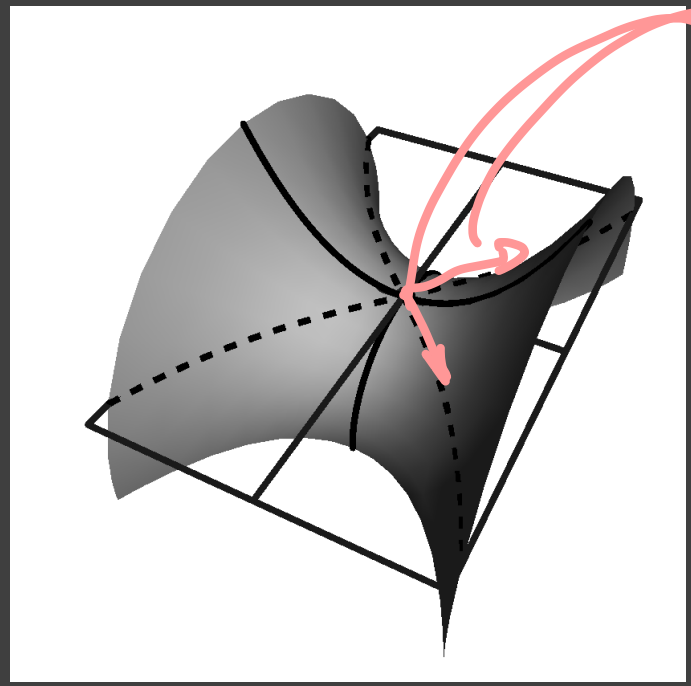
Q: I think I don't really understand why there is mention of asymptoticism. Why are the name introduced in this lecture labelled "asymptotic" parameter?

$\Updownarrow$

$II = 2M\, d\xi\, d\eta$

Asymtotic directions

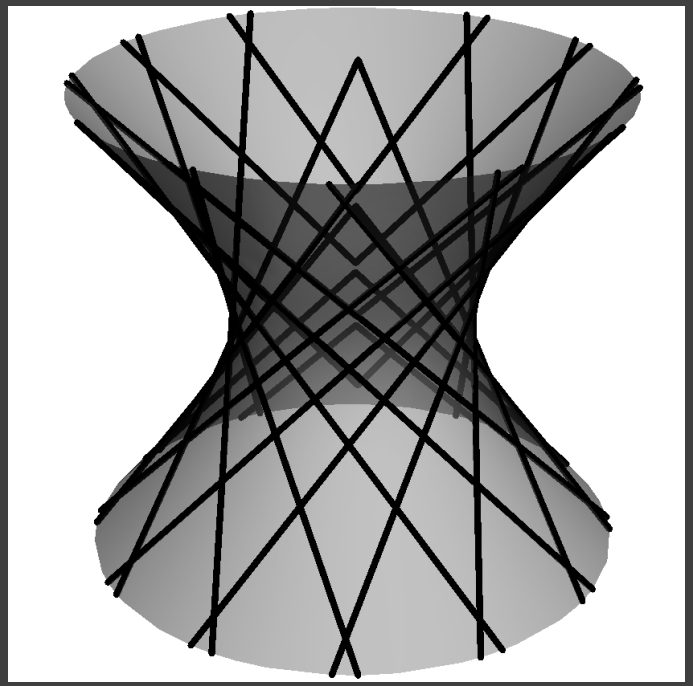




$K > 0$ $K < 0$

Df: Asymptotic parameters: coordinate curves $u \longmapsto p(u, v_0)$, $v \longmapsto p(u_0, v)$ are tangential to asymptotic directions at all points

Asymtotic parameters



1 sheeted hyperboloid

$x^2 + y^2 - z^2 = 1$

- asymptotic coord $\Leftrightarrow$ $\mathrm{II} = 2M\,du\,dv$.

Asymtotic parameters