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Kotaro Yamada
kotaro@math.sci.isct.ac.jp

Info. Sheet 4; Advanced Topics in Geometry B1 (MTH.B406)

Corrections

- Blackboard B, the second fundamental form in Exercise 2-2:

$$\frac{-1}{\sqrt{a^2+b^2}}\operatorname{sech} v \tanh v(a^2\,du^2 - 2ab\,du\,dv + b^2\,dv^2)$$
$$\Rightarrow \quad \frac{-1}{\sqrt{a^2+b^2}}\operatorname{sech} v \tanh v(a^2\,du^2 - 2ab\,du\,dv - a^2\,dv^2)$$

- Lecture Note, page 9, line 10: $(\xi^3, y) \Rightarrow (\xi^3, \eta)$
- Lecture Note, page 9, line 12:

$$D\varphi = \begin{pmatrix} x_\xi & x_\eta \\ y_\xi & y_\eta \end{pmatrix} = \begin{pmatrix} 2\xi^2 & 0 \\ 0 & 1 \end{pmatrix} \quad \Rightarrow \quad D\varphi = \begin{pmatrix} x_\xi & x_\eta \\ y_\xi & y_\eta \end{pmatrix} = \begin{pmatrix} 3\xi^2 & 0 \\ 0 & 1 \end{pmatrix}$$

- Lecture Note, page 9, line 19: $\varphi V \to U \Rightarrow \varphi\colon V \to U$
- Lecture Note, page 10, Lemma 3.5: P **and** $Q \Rightarrow \alpha$ and β
- Lecture Note, page 10, lines 21–22: the ordinary differential equation $\Rightarrow$ a system of ordinary differential equations
- Lecture Note, page 11, line 11: $v = \sqrt{1}2(s+t) \Rightarrow v = \frac{1}{\sqrt{2}}(s+t)$
- Lecture Note, page 11, line 12: $2M\,dt^2 \Rightarrow M\,dt^2$
- Lecture Note, page 12, Remark 3.11: The parameter $\Rightarrow$
  The parameter (ξ, η) as in (3.3) is called the *asymptotic Chebyshev net*.

Students' comments

- 計算が煩雑になってしまい，問題を解ききることができませんでした.
  The calculations became too complicated and I could not solve all the problems.

  Lecturer's comment Sorry.

Q 1: $0 < \theta(\xi, \eta) < \pi$ なのはなぜですか？
Why do we assume $0 < \theta(\xi, \eta) < \pi$?

A: To gurantee that the surface is a regular surface, i.e., p_ξ and p_η are linearly independent, or, equivalently, the determinant of the first fundamental matrix is positive.

Q 2: I think I don't really understand why there is mention of asymptoticism. Why are the name introduced in this lecture labelled "asymptotic" parameters?

A: Let P be a point on a surface where the Gaussian curvature is negative. Then it is known that the intersection of the tangent plane of the surface at P and the surface consists of tow curves intersecting at P on a neighborhood of P. We call the directions of two curves the **asymptotic direction**. An asymptotic coordinate system is a coordinate system whose each coordinate curve on the surface tangent to one of the asymtotic directions (cf. Section 9 and Appendix B-5 in [UY17]).