Advanced Topics in Geometry B1 (MTH.B406)

Kotaro Yamada
kotaro@math.sci.isct.ac.jp
http://www.official.kotaroy.com/class/2025/geom-b1

Institute of Science Tokyo

2025/07/11

Important notices

- Lecture on 18. July is cancelled. Next class will be 25th of July.
- Please fill the form “Course Survey” on LMS.

Q and A

Q: Is the definition of sine-Gordon equation $\theta_{xy} = \sin\theta$ or $\theta_{uu} - \theta_{vv} = \sin\theta$?

Both

Q: Is there any reason why constant negative Gaussian Curvature surfaces in the form of asymptotic Chebyshev net in (4.1) instead of (u, v) in (4.3) of the lecture note?

Q: Although I understand (u, v) to be just a parameter change from (x, y), switching this change the shape that the surface parametrized by p will take in the end, doesn't it? Then how aren't we straying from the Chebyshev net parametrization that we wanted to build from?

$$ds^2 = dx^2 + 2\cos\theta\, dx\,dy + dy^2 \quad \leftarrow \text{asymptotic Chebyshev} \qquad (4.1)$$

$$= \cos^2\frac{\theta}{2}\,du^2 + \sin^2\frac{\theta}{2}\,dv^2$$

$$\mathrm{II} = 2\sin\theta\, dx\,dy \quad \leftarrow \quad \sin\frac{\theta}{2}\cos\frac{\theta}{2}\left(du^2 - dv^2\right) \qquad (4.3)$$

- parameter change

$$x = \frac{1}{2}(u - v) \qquad y = \frac{1}{2}(u + v)$$

- essentially same

- Existence of coordinate systems
  - → 1st: the asymptotic Chebyshev
  - → 2nd: parameter change