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Info. Sheet 5; Advanced Topics in Geometry B1 (MTH.B406)

Informations

- Lecture on 18. July is cancelled. Next class will be 25th of July.
- Please fill the form "Course Survey" on LMS.

Corrections

- Lecture Note, page 13, line 10: parametrization ⇒ parametrization
- Lecture Note, page 14, bottom (eq. (4.14)): $\tanh v \Rightarrow \tanh \frac{v}{2}$
- Lecture Note, page 15, eq. (4.20)

$$dp = \cos\frac{\theta(v)}{2}\boldsymbol{v}_1(u)\,du + \frac{\dot\theta(v)}{2E}\sin\frac{\theta(v)}{2}\boldsymbol{v}_2(u)\,dv + \frac{1}{E}\sin\frac{\theta(v)}{2}\boldsymbol{v}_3\,dv.$$

$$\Rightarrow \qquad dp = \cos\frac{\theta(v)}{2}\boldsymbol{v}_1(u)\,du + \frac{\dot\theta(v)}{2E}\sin\frac{\theta(v)}{2}\boldsymbol{v}_2(u)\,dv + \frac{1}{E}\sin^2\frac{\theta(v)}{2}\boldsymbol{v}_3\,dv.$$

- Lecture Note, page 15, line −2:

$$p = \frac{-2}{E}\cos\frac{\theta}{2}\boldsymbol{v}_2 + \frac{1}{E}\boldsymbol{v}_3\int_{v_0}^{v}\sin\frac{\theta(t)}{2}\,dt,$$

$$\to \qquad p = \frac{-1}{E}\cos\frac{\theta}{2}\boldsymbol{v}_2 + \frac{1}{E}\boldsymbol{v}_3\int_{v_0}^{v}\sin^2\frac{\theta(t)}{2}\,dt,$$

- Lecture Note, page 16, line 4:
  (??) and resulting ⇒ the equation in Proposition 4.1 and resulting

Q 1: Sine Gordon equation の定義が $\theta_{xy} = \sin\theta$ なのか $\theta_{uu} - \theta_{vv} = \sin\theta$ なのかどちらでしょうか？
Is the definition of sine-Gordon equation $\theta_{xy} = \sin\theta$ or $\theta_{uu} - \theta_{vv} = \sin\theta$?

A: These are the same because they are transferred by a coordinate transformation. Both are called the Sine-Gordon equation.

Q 2: Gaussian Curvature が負の定数の表面（原文ママ：この講義の文脈では surface の訳語は「曲面」）を通常 Lecture note の (4.3) の (u, v) の形ではなく (4.1) の asymptotic Chebyshev net の形で表すのにはなにか理由がありますか？
Is there any reason why constant negative Gaussian Curvature surfaces in the form of asymptotic Chebyshev net in (4.1) instead of (u, v) in (4.3) of the lecture note?

A: We use the asymptotic Chebyshev net to show the existence of a coordinate system of the form (4.3). It might be possible to prove the existence of the coordinate system without going through this path.

Q 3: Exercise 4-2 は θ が C^2-級 (or C^∞-級) であれば十分だと思うのですが，$\theta_{xy} = \sin\theta$ を仮定しているのは sine-Gordon equation を満たしている θ から新たな φ を見つけられることを強調したいからなのでしょうか.
For Exercise 4-2, I think it is enough if θ is of class C^2 (or C^∞), but you assume $\theta_{xy} = \sin\theta$. Is it because you want to emphasize that we can find a new φ from θ that satisfies the sine-Gordon equation?

A: Yes. It is not necessary to assume $\theta_{xy} = \sin\theta$, but as a conclusion, we obtain it.

Q 4: This might be an unclever question, but although I understand (u, v) to be just a parameter change from (x, y), switching this change the shape that the surface parametrized by p will take in the end, doesn't it? Then how aren't we straying from the Chebyshev net parametrization that we wanted to build from?

A: I'm not sure but to show the existence of the parameter (u, v) as in (4.3), we need the existence of the asymptotic Chebyshev net (maybe...).