Advanced Topics in Geometry B1 (MTH.B406)

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Important notices

▶ ~~Lecture on 18. July is cancelled. Next class will be 25th of July.~~
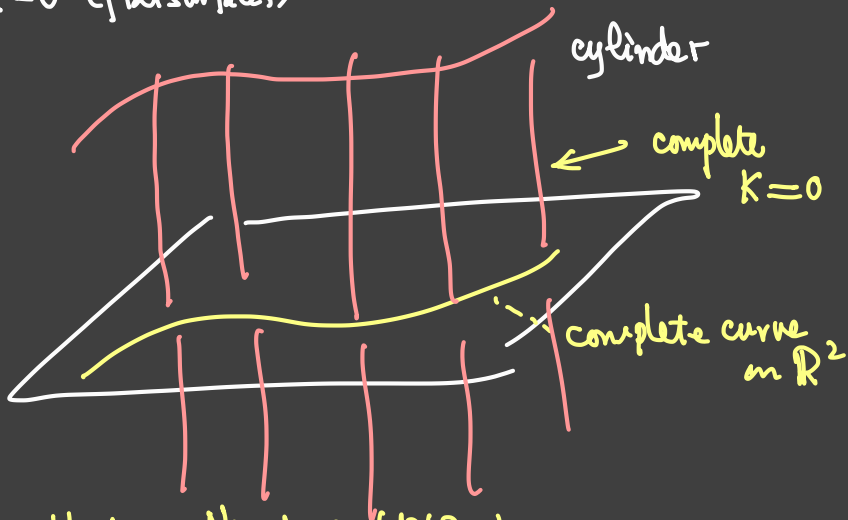
▶ Please fill the form "Course Survey" on LMS.

Q: Are there any other examples, such as pseudospherical surfaces, where the Gaussian curvature can tell if it is complete or not?

Hilbert's
thm
↙

- $\nexists$ complete surface in $\mathbb{R}^3$ with const $K < 0$.

- const $K > 0$ case: The only complete surface in $\mathbb{R}^3$ is the round sphere.

Liebman 1900
Compact ⟹ round sphere
Hartman - Nirenberg 1959,
complete ⟹ diffeo. S^2
compact

K=0 (flat surfaces)

cylinder

complete
K=0

complete curve
on $\mathbb{R}^2$

- Hartman-Nirenberg (1959)
  The only complete flat surfaces in $\mathbb{R}^3$
  are cylinders over complete curves.

Q and A

(Q: Is there a specific metric that is commonly used? Or
  are they prepared individually as needed?

- <u>Depending</u>.

- Some problem requires "good" metric

  const curvature, symmetric space ---

- Some problem looks for "good" metric.

  const curvature, Kähler Einstein ~~.