Advanced Topics in Geometry B1 (MTH.B406)

Lorentz-Minkowski space

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Exercise 5-1

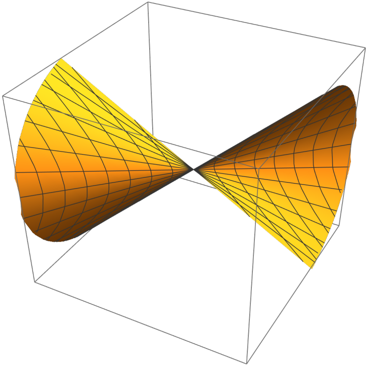
Problem

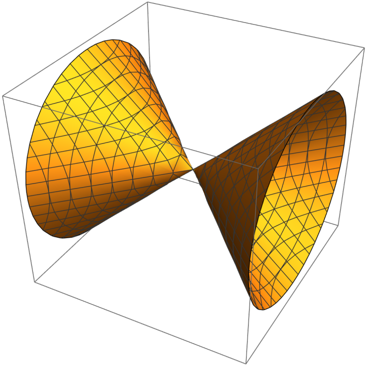
Consider a map

$$p\colon \mathbb{R}^2 \ni (u,v) \longmapsto (v\cosh u, v, v\sinh u) \in \mathbb{R}^3.$$

1. *Verify that the image* $p(\mathbb{R}^2)$ *is contained in the cone* $\{(x,y,z) \in \mathbb{R}^3\,;\, x^2 - y^2 - z^2 = 0\}$.
2. *Is the induced metric* $p^*\langle\ ,\ \rangle$ *complete on* $\mathbb{R}^2$*?*

Exercise 5-1





Exercise 5-2

Problem

Prove that the shortest curve (with respect to the canonical Riemannian metric) joining $\mathrm{O} := (0,0)$ *and* $\mathrm{P} := (L,0)$ *(*$L > 0$*) on the Euclidean plane is the line segment joining them.*

Recall

Problem (Exercise 2-1)

Let $\gamma(t) = \bigl(x(t), z(t)\bigr)$ ($\gamma \in I$) be a parametrized curve on the xz-plane satisfying

$$\bigl(x'(t)\bigr)^2 + (z'(t))^2 = 1 \qquad (t \in I), \tag{$*$}$$

where $I \subset \mathbb{R}$ is an interval. Consider a surface

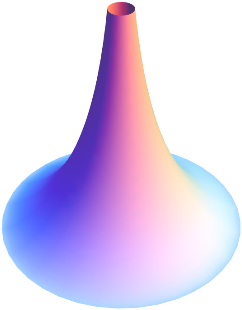
$$p_\gamma(s,t) := \bigl(x(t)\cos s, x(t)\sin s, z(t)\bigr),$$

which is a surface of revolution of profile curve γ.

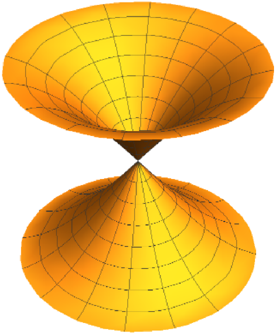
1. *Show that p_γ is pseudospherical if and only if $x'' = x$ holds.*
2. *Can one choose $I = \mathbb{R}$?*

Exercise 2-1

$x = e^{-s}$



$x = A \sinh s$



$x = A \cosh s$