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Info. Sheet 6; Advanced Topics in Geometry B1 (MTH.B406)

Informations

- Please fill the form "Course Survey" on LMS.

Corrections

- Lecture Note, page 17, lines 14–15: linear independent ⇒ linearly independent
- Lecture Note, page 17, lines 19–20: $ds \Rightarrow ds^2$ (3 times)
- Lecture Note, page 18, line 21: $\log|y_n| \Rightarrow |\log y_n|$
- Lecture Note, page 18, line −8: ***Hibert's*** ⇒ ***Hilbert's***
- *Lecture Note, page 19, line −12:* $\sin\theta_{xy} \Rightarrow \theta_{xy}$
- *Lecture Note, page 19, lines 1–2: a domain a domain* ⇒ *a domain*
- *Lecture Note, page 19, line 21:* $\int_{y_1}^{y^2} \Rightarrow \int_{y_1}^{y_2}$
- *Lecture Note, page 20, line 1:* $\varepsilon := \theta(x_2,0)-\theta(x_1,0) \Rightarrow \varepsilon := \theta(x_1,0)-\theta(x_2,0)$
- *Lecture Note, page 20, line 6:* $\theta(x,0)-\theta(0,0) > \theta(x_3,0)-\theta(0,0) \Rightarrow \theta(x,0)-\theta(0,0) \geqq \theta(x_3,0)-\theta(0,0)$
- *Lecture Note, page 20, line 15:* $\int_{x_2}^{x^1} \Rightarrow \int_{x_2}^{x_1}$

Students' comments

- Hilbert の定理が $\theta_{xy} = \sin\theta$ の性質から示せることに驚きました.
  I was surprised that Hilbert's theorem can be shown from the property $\theta_{xy} = \sin\theta$.

  Lecturer's comment Indeed.

Q 1: pseudospherical surface のようにガウス曲率によって完備かどうか分かる例は他にありますか？
Are there any other examples, such as pseudospherical surfaces, where the Gaussian curvature can tell if it is complete or not?

A: The only complete surface in $\mathbb{R}^3$ with positive constant Gaussian curvature is the round sphere (Liebmann, 1900, Hartman-Nirenberg, 1959), and the only complete flat surface ($K = 0$) is a cylinder over a complete curve of a plane (Hartman-Nirenberg, 1959).

Q 2: 一般的によく使われる計量は大体決まっているのでしょうか．それとも個別に適宜用意するのでしょうか.
Is there a specific metric that is commonly used? Or are they prepared individually as needed?

A: Depending on the problem.

Q 3: I suppose we can visualize the incompleteness of a surface the same way as that of metric space, meaning, a surface is complete if it has no missing point? Then, would it also be possible to construct a completion of a given non-complete manifold?

A: You're right, incompleteness is the existence of missing points, intuitively. and for incomplete manifold, there is a completion as a metric space.