Advanced Topics in Geometry B1 (MTH.B406)

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Important notices

- Today's lecture is the last one.
  Thank you for joining the class. I appreciate your feedback and comments which will be useful for my future lectures.

- Please fill the form "Course Survey" on LMS.

Q and A

$\mathbb{R}^{n+1}$

light-like vector

Q: Why does the fact that the light-like line $\gamma(s) = x + sv$ is invariant under Lorentz transformation, that is, invariant under different observers, represent the principle of light speed invariance? I don't understand the relationship between $\gamma(s)$ and the speed of light. $\bar{x}$

light-like

$\bar{v}$

$$A\gamma(s) + a = (Ax + a) + sAv$$

$$A \in O(n,1)$$

$n=3$

$$v = (v_0 \ v_1 \ v_2 \ v_3)^T \text{ is light-like}$$

$$\Leftrightarrow v_0^2 = v_1^2 + v_2^2 + v_3^2 \ : \text{velocity vector}$$

$\mathbb{L}^4 \supset \mathbb{R}^3 = \{(0, x_1, x_2, x_3) ; x_j \in \mathbb{R}\}$

$\longrightarrow$ "space"

x_0: time

$\pi \circ \gamma =$ motion of "points" on the space $\mathbb{R}^3$

$\downarrow$

speed in $\mathbb{R}^3$ is $\dfrac{\sqrt{v_1^2 + v_2^2 + v_3^2}}{(v_0)^2}$

≤ 1

invariant under Lorentz transf.

Q and A

Q: Does the hyperbolic space have a physical interpretation to it? As I understand it is a subspace of the Lorentz-Minkowski space where all vectors are time-like of a particular kind ($\langle x, x\rangle = -1$).

$$H^n = \{x \in \mathbb{L}^{n+1} ; \langle x, x\rangle = -1, x_0 > 0\}$$

$= $ "time-like unit vectors"

$$S^n = \{x \in \mathbb{R}^{n+1} ; x \cdot x = 1\}$$

$= $ "unit vectors in $\mathbb{R}^{n+1}$"