August 1, 2025
Kotaro Yamada
kotaro@math.sci.isct.ac.jp

Info. Sheet 7; Advanced Topics in Geometry B1 (MTH.B406)

Informations

- Today's lecture is the last one. Thank you for joining the class. I appreciate your feedback and comments which will be useful for my future lectures.

- Please fill the form "Course Survey" on LMS.

Corrections

- Handout C, page 5: consits of for connected components ⇒ consists of four connected components
- Lecture Note, page 21, line 6: consits ⇒ consists
- Lecture Note, page 22, line −8––7: this curve is also represents ⇒ this curve also represents
- Lecture Note, page 21, line −2: **isotoropic ⇒ isotropic**
- Lecture Note, page 24, line 5: elenents ⇒ elements
- Lecture Note, page 24, line −2: Let $\boldsymbol{x}$ and $\boldsymbol{y}$ be two distinct points of S^n with $\boldsymbol{y} \neq -\boldsymbol{x}$.

Q 1: Exer 6-1 って解けますか？ 私の誤りの可能性は大きいですが，固有値が一致ということから必要条件を求めて進めたところ，A, B は共役にならなかった.
Can you solve Exer 6-1? There is a possibility that I am wrong, but I proceeded to find the necessary condition from the fact that the eigenvalues coincide, and A and B are not conjugate.

A: You are right. The answer is "never".

Q 2: light-like line $\gamma(s) = -\boldsymbol{x} + s\boldsymbol{v}$ がローレンツ変換で不変，つまり観測者を変えても不変であることが，なぜ光速度不変の原理を表すのですか．$\gamma(s)$ と光速度の関係がよくわかりません.
Why does the fact that the light-like line $\gamma(s) = -\boldsymbol{x} + s\boldsymbol{v}$ is invariant under Lorentz transformation, that is, invariant under different observers, represent the principle of light speed invariance? I don't understand the relationship between $\gamma(s)$ and the speed of light.

A: When $\gamma(t)$ is regarded as the world line of a point, its x_0-component can be regarded as the time and (x_1, x_2, x_3)-component as the coordinates of "space". Therefore, the speed of a point in the corresponding space is $\sqrt{v_1^2 + v_2^2 + v_3^2}/|v_0|$, where $\boldsymbol{v} = (v_0, v_1, v_2, v_3)^T$. In particular, a necessary and sufficient condition for $\boldsymbol{v}$ to be light-like is the speed 1 (=light-speed). This property is invariant under Lorentz transformations.

Q 3: 「A is conjugate to B」というのは「ある正則行列 P が存在して $A = P^{-1}AP$ となる」という意味でとらえたのですが，別の意味があるのでしょうか.
I took "A is conjugate to B" to mean "there exists a regular matrix P such that $A = P^{-1}AP$", but is there another meaning?

A: No.

Q 4: Does the hyperbolic space have a physical interpretation to it? As I understand it is a subspace of the Lorentz-Minkowski space where all vectors are time-like of a particular kind ($\langle \boldsymbol{x}, \boldsymbol{x} \rangle = -1$.

A: The (unit) sphere $S^n = \{\boldsymbol{x} \in \mathbb{R}^{n+1} \, ; \, |\boldsymbol{x}| = 1\}$ is regarded as the set of unit vectors in the Euclidean space. Similarly, the hyperbolic space is the set of "unit" time-like vectors in the Lorentz-Minkowski space.