⊺ I T L E & A B S T R A C T

The 7th Geometry Conference for the Friendship of Japan and China

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Tokyo & Lake Kawaguchi

Bochner Type Inequality on Alexandrov Spaces

Xi-Ping Zhu (Sun Yat-sen University)

【Abstract】On an Alexandrov space, one has the concept of first deivative for functions. But it is difficult to define second derivatives. In the talk, I will discuss how to overcome the difficulty to establish a Bochner type inequality on Alexandrov spaces. As consequences, we obtained several rigidity theorems such as Cheeger-Gromoll type splitting theorem, Cheng type maximal diameter theorem and Obata type theorem on Alexandrov spaces. We also used the Bochner type inequality to derive Yau's gradient estimates, sharp first eigenvale estimate and Li-Yau estimate on Alexandrov spaces. This is a joint work with Hui-Chun Zhang.

Affine techniques and Extremal metrics on toric manifolds

An-Min Li (Sichuan University)

【Abstract】We will explain the affine techniques and their applications in the study of extremal metrics on toric manifolds.

Curvature and Fundamental Groups

Xiaochun Rong (Rutgers University/Capital Normal University)

Parametrization and Optimization of test configurations

Yuji Odaka (Kyoto University, RIMS)

【Abstract】We will show that, roughly speaking, Tit's spherical building gives a nice parametrization of test configurations and prove the existence of ``maximally destabilizing test configurations" for Chow-unstable varieties. We may also make other remarks on test configurations if time permits.

Uniform stability of extremal metrics on toric varieties

Bohui Chen (Sichuan Univeristy)

【Abstract】 In this talk, I will explain a uniform stability of extremal metrics on toric varieties. This is a joint work with An-Min Li and Li Sheng. We show that such a stability is a necessary condition for extremal metrics.

Self-shrinkers of the Mean Curvature Flow in Arbitrary Codimension

Haizhong Li (Tsinghua University)

【Abstract】 In this talk, we will present some typical examples and classification results of self-shrinkers of the mean curvature flow in arbitrary codimension. In particular , we will present a classification theorem for self-shrinkers of the mean curvature flow with $|A|^2\leq 1$ in arbitrary codimension (joint with Huaidong Cao), which implies a gap theorem for self-shrinkers in arbitrary codimension.

[CL] Huai-Dong Cao, Haizhong Li, A Gap Theorem for Self-shrinkers of the Mean Curvature Flow in Arbitrary Codimension}, arXiv: 1101.0516v1

Some convexity estimates for the solutions elliptic partial differential equations

Xinan MA (University of Science and Technology of China)

【Abstract】 We study the convexity estimates for the solutions or the level sets of the solution elliptic partial differential equations.

On Perelman's entropy for the Witten-Laplacian and the Fokker-Planck equation on Riemannian manifolds

Xiangdong Li (Chinese Academy of Sciences)

【Abstract】 Inspired by Perelman's entropy formula for Ricci flow, we study the Perelman W-entropy for the heat equation of the Witten-Laplacian and the Fokker-Planck equation on Riemannian manifolds. Under some natural geometric condition in terms of the Bakry-Emery Ricci curvature, we prove the monotonicity theorem and a rigidity theorem of the Perelman W-entropy on complete Riemannian manifolds.

Deformations of the Killing spinor equation on Sasaki-Einstein and 3-Sasaki manifolds

Craig van Coevering (Max Planck Institute for Mathematics, Bonn/USTC)

【Abstract】 see Attachment (1)

Recent progress on the classification of non-degenerate equiaffine hyperspheres

Zejun Hu (Zhengzhou University)

【Abstract】see Attachment (2)

Obstruction-Flat Asymptotically Locally Euclidean Metrics

Jeff Viaclovsky (University of Wisconsin)

【Abstract】 Given an even dimensional Riemannian manifold (M^{n}, g) with $n \geq 4$, it was shown in the work of Charles Fefferman and Robin Graham on conformal invariants the existence of a non-trivial 2-tensor which involves n derivatives of the metric, and arises as the first variation of a conformally invariant functional and vanishes for metrics that are conformally Einstein. This tensor is called the Ambient Obstruction tensor and is a higher dimensional generalization of the Bach tensor in dimension 4. We show that any asymptotically locally Euclidean (ALE) metric which is obstruction flat and scalar-flat must be ALE of a certain optimal order using a technique developed by Cheeger and Tian for Ricci-flat metrics. We also prove a singularity removal theorem for obstruction-flat metrics with isolated C^{0}-orbifold singularities. In addition, we show that our methods apply to more general systems. This is joint work with Antonio Ache.

Construction of calibrated submanifolds

Kotaro Kawai (Tohoku University)

【Abstract】The notion of calibrated submanifolds was introduced by Harvey and Lawson in 1982. They are important in mirror symmetry due to the SYZ conjecture. In this talk, I will construct special Lagrangian submanifolds in non-flat Calabi-Yau manifolds explicitly. I also consider the coassociative submanifolds in G_2 manifolds in a similar way.

The Abreu equation with degenerated boundary conditions

Li Sheng (Sichuan University)

【Abstract】The Abreu equation is a fully nonlinear 4th order partial differential equation that arises from the study of the extremal metrics on toric manifolds. We study the Dirichlet problem of the Abreu equation with degenerated boundary conditions. The solutions provide the Kahler metrics of constant scalar curvature on the complex torus.

Bounded Harmonic Functions on Riemannian Manifolds of Nonpositive Curvature

Qing Ding (Fudan University)

【Abstract】Certain general conditions are put forth on a complete simply-connected Riemannian manifold of nonpositive curvature which guarantee that they support nontrivial bounded harmonic functions. This result includes the Cartan-Hadamard manifolds with curvature pinched between two negative constants and the bounded symmetric domains ${\mathfrak R}_I(n,n)$ and ${\mathfrak R}_{II}(n)$ ($n\ge2$) as special cases.

Compactness for almost Kahler-Einstein manifolds

Gang Tian (Princeton University/Peking University)

Brody curves and mean dimension

Masaki Tsukamoto (Kyoto University)

【Abstract】Brody curve is a 1-Lipschitz holomorphic map from the complex plane to the projective space. The space of Brody curves admits a natural group action, and it becomes an infinite dimension dynamical system. We study the mean dimension of this dynamical system. The main result is the formula of the mean dimension of the system of Brody curves in the Riemann sphere. This is the joint work with Shinichiroh Matsuo (Kyoto).

Variational solutions to extremal metrics on toric surfaces

Bin Zhou (Peking University)

【Abstract】In this talk, I will talk about the interior regularity of minimizers of Donaldson's version of K-energy associated to Calabi's extremal metrics on toric surfaces. A new idea is to use a sequence of solutions to the first boundary value problem of Abreu's equation to approximate a minimizer of K-energy. By the regularity, we also prove the uniqueness of minimizers in dimension 2.

Isometric embedding of negatively curved surfaces

Bing-Long Chen (Sun Yat-Sen University)

【Abstract】Hilbert-Efimov theorem states that any complete surface with curvature bounded above by a negative constant can not be isometrically embedded in three dimensional Euclidean space ${R}^3$. We demonstrate that any simply-connected smooth complete surface with curvature bounded above by a negative constant admits a global smooth isometric embedding into the Lorentz-Minkowski space ${R}^{2,1}$.

Fibred toric varieties in toric hyperkahler varieties

Wei Zhang (University of Science and Technology of China)

【Abstract】We introduce the fibred toric varieties as equivariant CP^r bundles over lower dimensional toric varieties. An equivalent characterization is that some natural morphisms on them degenerate to bundle projections in the context of variation of toric varieties as GIT quotients. As an application, our main observation is that these fibred toric varieties also arise naturally in the variation of hyperkahler varieties, namely, the fibred toric varieties are contained in the exceptional sets of the hyperkahler natural morphisms and the Mukai flops.

Extrinsic diameter of immersed flat tori in 3-sphere

Masaaki Umehara (Tokyo Institute of Technology)

【Abstract】Enomoto, Weiner and Kitagawa showed the rigidity of the Clifford torus amongst the class of embedded flat tori in the 3-sphere. In the proof of that result, an estimate of extrinsic diameter of flat tori plays a crucial role. It is reasonable to expect that the same rigidity holds in the class of immersed flat tori in the 3-sphere. In this talk, we show that the Clifford torus is rigid in the class of immersed flat tori whose mean curvature functions do not change sign. This is a joint work with Yoshihisa Kitagawa (Utsumomiya Univ.)

Dirac operators and vanishing theorems

Weiping Zhang (Nankai University)

【Abstract】We will describe a few results from the the vast development arised from two classical vanishing theorems, due to Lichnerowicz and Atiyah-Hirzebruch respectively, involving Dirac operators.

Convergence of Kahler to real polarizations on flag manifolds

Hiroshi Konno (University of Tokyo)

【Abstract】In this talk we will discuss geometric quantization of a flag manifold. In particular, we construct a family of complex structures on a flag manifold that converge 'at the quantum level' to the real polarization coming from the Gelfand-Cetlin integrable system. Our construction is based on a toric degeneration of flag varieties and a deformation of Kahler structures on toric varieties by symplectic potentials. This is a joint work with Mark Hamilton.

A dynamical approach to balanced metrics with automorphism group

Yuji Sano (Kumamoto University)

【Abstract】Balanced metrics on a polarized manifold can be considered as a fixed point of some discrete dynamical system on the space of Kahler metrics (or Bergman metrics) introduced by Donaldson. I try to extend this picture to the case where the automorphism group is not discrete.

On the convergence of a Kahler-Ricci flow

Zhenlei Zhang (Capital Normal University)

【Abstract】Let (M,J) be a compact Kahler manifold with positive first Chern class c_1 and X be a holomorphic vector field. I will show that, if the modified Futaki invariant with respect to X vanishes and a Kahler metric $\omega \in 2\pi c_1$ is sufficiently close to a shrinking Ricci soliton, then the Kahler-Ricci flow will really converges to the shrinking Ricci soliton. I will also try to give applications.

Lagrangian Floer theory of arbitrary genus and Gromov-Witten invariant

Kenji Fukaya ((Kyoto University)

【Abstract】Given a finitely many collection of unobstructed Lagrangian submaifolds, we consider the symmetric bar complex of the cyclic barcomplex of Floer complex. It has a structure of differential involutive bi-Lie algebra (dIBL) via intersection pairing and Floer's boundary operator (joint work with Cielibak and Latshev). The Lagrangian Floer theory of arbitrary genus can be formulated so that it gives a solution of dIBL version of Mauren-Cartan equation (BV Master equation) of this dIBL algebra. The main result I want to explain in this talk is that in case if this set of Lagrangian submaifold is big enough, there is a formula to calculate Gromov-Witten invariant (of arbitrary genus) of ambient manifold from this Mauren-Cartan element.

Deformations of the Killing spinor equation on Sasaki-Einstein and 3-Sasaki manifolds

The seventh Geometry Conference for the Friendship of Japan and China, January 2012

Craig van Coevering

A simply connected Sasaki-Einstein manifold has 2 linearly independent Killing spinors while a 3-Sasaki manifold of dimension $4m - 1$ has $m + 1$ linearly independent Killing spinors. Infinitesimal transversal holomorphic deformations of the Reeb foliation $\mathscr{F}_\xi$, elements of $H^1_{\bar{\partial}_b}(\Theta_{\mathscr{F}_\xi})$, where $\Theta_{\mathscr{F}_\xi}$ is the sheaf of transversely holomorphic vector fields, give rise to infinitesimal Einstein deformations. We show that under these infinitesimal Einstein deformations the 2 Killing spinors in the first case are preserved to first order, while the remaining $m - 1$ in the 3-Sasaki case are never preserved.

While the question of integrability of these infinitesimal Einstein deformations in the Sasaki-Einstein case is unanswerable in general, we prove that in the 3-Sasaki case the real subspace $\mathrm{Re}\, H^1_{\bar{\partial}_b}(\Theta_{\mathscr{F}_\xi})$ integrates to actual Sasaki-Einstein deformations, i.e. preserving precisely a 2 dimensional subspace of Killing spinors. We consider this in the case of toric 3-Sasaki 7-manifolds. The underlying Sasaki-Einstein structure of a toric 3-Sasaki 7-manifold M has a real $b_2(M) - 1$ dimensional space of Sasaki-Einstein deformations, where $b_2(M)$ can be arbitrarily large. Thus we have examples of Einstein 7-manifolds admitting 3 Killing spinors which have a family of deformations to Einstein metrics admitting only 2 Killing spinors. Therefore, unlike the case of parallel spinors, the dimension of the space of Killing spinors does not always remain constant under Einstein deformations.

Recent progress on the classification of non-degenerate equiaffine hyperspheres

Zejun Hu

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Let M be a non-degenerate hypersurface in the real $(n+1)$-dimensional unimodular-affine space $\mathbb{R}^{n+1}$ equipped with its canonical flat connection D and a parallel volume form. Let ξ be the equiaffine normal, h its affine Blaschke-Berwald metric, ∇ its induced affine connection, S its shape operator. Then we have

$$D_X Y = \nabla_X Y + h(X,Y)\xi, \quad D_X \xi = -S(X).$$

The hypersurface M is called an affine hypersphere if and only if $S = \lambda \operatorname{id}$. Moreover, if $\lambda = 0$, all affine normals are parallel and M is called a parabolic affine hypersphere. If $\lambda \neq 0$, all affine normals pass through a fixed point. In the latter case, if the metric h is positive definite, we call M an elliptic affine hypersphere if $\lambda > 0$ and an hyperbolic affine hypersphere if $\lambda < 0$.

The fundamental theorem of equiaffine hypersurfaces differential geometry states that h and C uniquely determine a non-degenerate hypersurface M in $\mathbb{R}^{n+1}$ up to equiaffine equivalence. Moreover, the classical Blaschke-Pick-Berwald theorem shows that $C \equiv 0$ on M if and only if M is an open part of a non-degenerate quadric.

On the other hand, the next natural problem on the classification of non-degenerate hypersurfaces with $\hat{\nabla}C \equiv 0$, *which is a strong condition and it implies that the hypersurface should be an affine homogeneous hypersphere*, turns out very complicated. If h is definite (i.e., M is locally strongly convex), after many years' continuous efforts by Dillen, Vrancken, et al. and obtaining the classifications for lower dimensions, very recently Hu, Li and Vrancken eventually established the complete classification as below:

Classification Theorem (Z. Hu-H. Li-L. Vrancken, J. Diff. Geom., 2011)
Let M^n be an n-dimensional ($n \geq 2$) locally strongly convex affine hypersurface in $\mathbb{R}^{n+1}$ with $\hat{\nabla}C = 0$. Then M is a quadric (i.e. $C = 0$) or a hyperbolic affine hypersphere with $C \neq 0$, in the latter case either

(i) *M^n is obtained as the Calabi product of a lower dimensional hyperbolic affine hypersphere with parallel cubic form and a point, or*

(ii) *M^n is obtained as the Calabi product of two lower dimensional hyperbolic affine hyperspheres with parallel cubic form, or*

(iii) $n = \frac{1}{2}m(m+1) - 1$, $m \geq 3$, and (M^n, h) is isometric with $\mathbf{SL}(m, \mathbb{R})/\mathbf{SO}(m)$, and the immersion is affinely equivalent to the standard embedding of $\mathbf{SL}(m, \mathbb{R})/\mathbf{SO}(m) \hookrightarrow \mathbb{R}^{n+1}$, or

(iv) $n = m^2 - 1$, $m \geq 3$, and (M^n, h) is isometric with $\mathbf{SL}(m, \mathbb{C})/\mathbf{SU}(m)$, and the immersion is affinely equivalent to the standard embedding of $\mathbf{SL}(m, \mathbb{C})/\mathbf{SU}(m) \hookrightarrow \mathbb{R}^{m^2}$, or

(v) $n = 2m^2 - m - 1$, $m \geq 3$, and (M^n, h) is isometric with $\mathbf{SU}^*(2m)/\mathbf{Sp}(m)$, and the immersion is affinely equivalent to the standard embedding of $\mathbf{SU}^*(2m)/\mathbf{Sp}(m) \hookrightarrow \mathbb{R}^{n+1}$, or

(vi) $n = 26$ and (M^{26}, h) is isometric with $\mathbf{E}_{6(-26)}/\mathbf{F}_4$, and the immersion is affinely equivalent to the standard embedding of $\mathbf{E}_{6(-26)}/\mathbf{F}_4 \hookrightarrow \mathbb{R}^{27}$.

Generally, it is well known that the class of affine hyperspheres is very large and that one is far from a classification; even under strong additional assumptions there only exist partial classifications. Apart from the above classification theorem, another famous problem is the *local classification of affine hyperspheres such that the metric h is of constant sectional curvature*, which up to present only partial answer is known.

In this lecture, I will talk about the recent progress on the classification of non-degenerate affine hyperspheres, mainly obtained by joint work of Cece Li, Haizhong Li, Luc Vrancken and the speaker.